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The intensity-fluctuation distribution of Gaussian light

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Abstract. We discuss the quantum-mechanical problem of intensity fluctuations in samples of Gaussian light and its connection with the corresponding classical scalar problem involving fluctuations of random noise power. We obtain an expression for the intensity-fluctuation distribution of light with a Lorentzian spectrum, and investigate analytically its asymptotic behaviour. Numerical techniques are used to evaluate the distribution for various intermediate linewidths, and some examples of the corresponding theoretical photon-counting distributions are also given.

1. Introduction

With the rapid progress now being made in laser technology it seems reasonable to believe that before long information about correlations of excitations in liquids and solids, particularly lifetimes (for instance of sound waves) will be obtainable by a study of the statistical (coherence) properties of scattered laser light. Such experiments have already been successful near critical phenomena where the scattering is very strong (Ford and Benedek 1965, Alpert *et al.* 1965). It will very often be the case that the scattering process is a purely random one and the scattered light will be Gaussian in character with a Lorentzian spectrum.

The statistical properties of the light are contained in the correlations of the randomly fluctuating values of, say, the electric field $\mathscr{E}(r, t)$ at different points of space and time. An ideal light detector at a point in space responds not to the electric field itself but to the modulus of the square of the positive-frequency part $\mathscr{E}^+(r, t)$, corresponding to annihilation of photons; it will also have a finite time constant T. The information obtainable from a single detector in this type of experiment is therefore contained in the statistical behaviour of the quantity

$$E(T) = \int_0^T \mathscr{E}^+(r, t)\mathscr{E}^-(r, t)\, dt \tag{1}$$

where we know that $\mathscr{E}(r, t)$ arises from a Gaussian light source with Lorentzian spectrum of given width Γ. In this paper we calculate the exact probability distribution of $E(T)$ for arbitrary values of ΓT.

The problem is a two-dimensional generalization of the scalar problem, considered in the classic papers of Rice (1944, 1945) and more completely solved later by Slepian (1958), of the fluctuations of random noise power of a sample of finite duration T of Gaussian noise $\mathscr{E}(t)$ with a given power spectrum. In this case the probability distribution of the quantity

$$E_c(T) = \int_0^T \mathscr{E}^2(t)\, dt$$

is found. This would be the intensity-fluctuation distribution of Gaussian light if the classical form $\mathscr{E}^2(t)$ were taken for the intensity instead of the quantum-mechanical formula. We shall show that the distributions of $E_c(T)$ and $E(T)$ are quite different. The classical formula has been used incorrectly in the past; we note, for instance, that equation (6.13) of the review paper by Mandel and Wolf (1965) is based on the classical instead of the quantum-mechanical distribution.

The moment generating function for the distribution of $E(T)$ has been obtained by Glauber (1965) in terms of its Fredholm determinant, but without explicit calculation of the eigenvalues. This determinant will be shown to be equivalent to the one appearing in Slepian's paper referred to above. Bédard (1966) has also obtained Slepian's form for a Fredholm determinant which, however, together with his integral equation and its eigenvalues differ somewhat from the equations to be obtained below. Since no reference is made to the relevant works of Glauber or Slepian it is not clear where these slight differences arise; there is no discrepancy, however, in the practical results.

In § 2 the moment generating function is obtained and a connection is made between the matrix relations of Glauber and the integral equation of Slepian. In § 3 the particular case of a Lorentzian spectrum is considered and an expression for $P(E)$ as the inverse Laplace transform of the generating function is found by summing residues. The distributions in the asymptotic limits of ΓT much greater than and much less than 1 are derived and an approximate form is given for $\Gamma T \sim 1$ in § 4. The main results of the paper are presented in graphical form in § 5.

2. The moment generating function

The positive-frequency part of the electric field may be expanded in normal modes

$$\mathscr{E}^{+}(r, t) = \sum_{k} \alpha_k e_k(r, t) \tag{2}$$

where

$$e_k(r, t) = i\{\tfrac{1}{2}\hbar\omega_k\}^{1/2} u_k(r) \exp(-i\omega_k t); \tag{3}$$

the notation is that of Glauber (1963).

The r dependence will be dropped henceforth as we are considering the field at a single point only. The α_k are, in general, random variables; for Gaussian light they are uncorrelated by definition and have the Gaussian distribution (Glauber 1963)

$$P(\alpha_k) = \frac{1}{\pi\langle n_k\rangle} \exp\left(-\frac{|\alpha_k|^2}{\langle n_k\rangle}\right) \tag{4}$$

where the values of $\langle n_k\rangle$ are given by

$$\langle \alpha_i^* \alpha_k\rangle = \langle n_k\rangle \delta_{ik}; \tag{5}$$

these are determined by the frequency spectrum of the light. For a Lorentzian spectrum centred on ω_0 with half-width at half-height Γ we have, for instance,

$$\langle n_k\rangle \hbar\omega_k \propto \frac{1}{(\omega_k - \omega_0)^2 + \Gamma^2}. \tag{6}$$

In terms of the normal modes the integrated intensity may be written, using equations (1) and (2),

$$E(T) = \sum_{kl} \int_0^T \alpha_l^* e_l^*(t) e_k(t) \alpha_k \, dt. \tag{7}$$

The e_k's are not orthogonal in the restricted interval $0 \leqslant t \leqslant T$ and the problem is simplified by introducing a new basis set $\phi_i(t)$, which is obtained by a unitary transformation S of the e_k's

$$e_k(t) = \sum_{i} S_{ki} \phi_i(t) \tag{8}$$

and which is complete and orthonormal in this interval

$$\int_0^T \phi_i(t) \phi_j(t) \, dt = \delta_{ij}. \tag{9}$$

Since there are infinitely many such sets we can require the transformation to have the further property that the coefficients a_i of $\mathscr{E}^{+}(t)$ in the basis ϕ_i are statistically independent.

Thus from equations (2), (8) and (9)

$$\int_0^T \mathscr{E}^+(t)\phi_i(t)\,dt = \sum_k \alpha_k S_{ki} = a_i \tag{10}$$

and

$$\langle a_i^* a_k \rangle = \langle m_k \rangle \delta_{ik}. \tag{11}$$

The integrated intensity in the new basis is found from equations (7), (8), (9) and (10):

$$\begin{aligned} E(T) &= \sum_{ijkl} a_l^* S_{li}^* S_{kj} \alpha_k \int_0^T \phi_i(t)\phi_j(t)\,dt \\ &= \sum_k |a_k|^2 \end{aligned} \tag{12}$$

and the linear nature of the transformation from α_k to a_k implies that the probability distributions of the a_k are of the form

$$P(a_k) = \frac{1}{\pi\langle m_k \rangle} \exp\left(-\frac{|a_k|^2}{\langle m_k \rangle}\right). \tag{13}$$

The moment generating function $Q(s)$ of $P(E)$ is thus

$$Q(s) = \langle e^{-sE} \rangle = \int \exp\left(-s\sum_j |a_j|^2\right) \prod_k \frac{1}{\pi\langle m_k \rangle} \exp\left(-\frac{|a_k|^2}{\langle m_k \rangle}\right) d^2 a_k. \tag{14}$$

To perform this integration we make the transformation

$$|a_k|^2 = \langle m_k \rangle |b_k|^2 \tag{15}$$

which gives, after integration over the angle variable,

$$\begin{aligned} Q(s) &= \int \prod_k \exp\{-|b_k|^2(1+s\langle m_k \rangle)\}\, d|b_k|^2 \\ &= \prod_k (1+s\langle m_k \rangle)^{-1}. \end{aligned} \tag{16}$$

We shall first show that the $\langle m_k \rangle$ are the eigenvalues of the matrix

$$M = \langle n_k \rangle^{1/2} \int_0^T e_k^*(t) e_{k'}(t)\,dt\, \langle n_{k'} \rangle^{1/2} \tag{17}$$

in order to make contact with Glauber's result and we shall then find an integral equation for the m_k's to compare with the work of Slepian. The matrix which diagonalizes M is S, for

$$\begin{aligned} S^\dagger M S|_{kk'} &= \int_0^T \sum_{ij} S_{ki}^\dagger \langle n_i \rangle^{1/2} e_i^*(t) e_j(t) \langle n_j \rangle^{1/2} S_{jk'}\,dt \\ &= \int_0^T \sum_{ijrs} \langle n_i \rangle^{1/2} S_{ki}^\dagger S_{ir}^* \phi_r^*(t) \phi_s(t) S_{js} S_{jk'} \langle n_j \rangle^{1/2}\,dt \end{aligned}$$

which, using equation (9) and the unitary properties of S, becomes

$$S^\dagger M S|_{kk'} = \sum_i \langle n_i \rangle S_{ik}^* S_{ik'} \tag{18}$$

and, using equations (5), (10) and (11), finally reduces to

$$S^\dagger M S|_{kk'} = \langle m_k \rangle \delta_{kk'}. \tag{19}$$

The integral equation for the $\langle m_k \rangle$ is obtained by squaring equation (10):

$$\int_0^T \int_0^T \mathscr{E}^+(t)\mathscr{E}^-(t')\phi_k(t)\phi_{k'}^*(t')\,dt\,dt' = a_k a_{k'}^*.$$

Taking the expectation value and using equation (11) gives

$$\int_0^T \int_0^T \langle \mathscr{E}^+(t)\mathscr{E}^-(t') \rangle \phi_k(t)\phi_{k'}^*(t')\,dt\,dt' = \langle m_k \rangle \delta_{kk'}. \tag{20}$$

We now define the first-order correlation function

$$G_1(t, t') = \langle \mathscr{E}^+(t)\mathscr{E}^-(t') \rangle \tag{21}$$

and multiply equation (20) by $\phi_{k'}(t'')$ and sum over k' to obtain

$$\int_0^T G_1(t, t'')\phi_k(t)\,dt = \langle m_k \rangle \phi(t''). \tag{22}$$

To compare this equation with the integral equation of Slepian's problem we divide both sides by $G_1(t, t)$:

$$G_1(t, t) = \langle \mathscr{E}^+(t)\mathscr{E}^-(t) \rangle = \langle E \rangle / T \tag{23}$$

where $\langle E \rangle$ is the expectation of E. The normalized first-order correlation function for Gaussian light with Lorentzian spectrum is (Glauber 1965)

$$g_1(t, t') = \frac{G_1(t, t')}{G_1(t, t)} = \exp\{i\omega_0(t-t') - \Gamma|t-t'|\} \tag{24}$$

so that (22) may be written

$$\int_0^T \exp\{i\omega_0(t-t') - \Gamma|t-t'|\}\phi_k(t)\,dt = \lambda_k \phi_k(t') \tag{25}$$

where we have defined the eigenvalues λ_k as

$$\lambda_k = \frac{\langle m_k \rangle T}{\langle E \rangle}. \tag{26}$$

On substituting

$$\phi_k(t)\exp(i\omega_0 t) = \Phi_k(t) \tag{27}$$

into (25) gives the required integral equation of Gaussian RC noise theory

$$\int_0^T \exp(-\Gamma|t-t'|)\Phi_k(t)\,dt = \lambda_k \Phi_k(t'). \tag{28}$$

The moment generating function of equation (16) may now be written, using equation (26),

$$Q(s) = \prod_k \left(1 + s\frac{\langle E \rangle}{T}\lambda_k\right)^{-1}. \tag{29}$$

This Fredholm determinant has been evaluated by Slepian (1958). To make a comparison we replace Slepian's variable t by $\langle E \rangle s/2$. With a little algebra we find

$$Q(s) = e^{\gamma}\left\{\cos y + \left(\frac{\gamma}{2y} - \frac{y}{2\gamma}\right)\sin y\right\}^{-1} \tag{30}$$

where

$$y = \{-(2\langle E \rangle \gamma s + \gamma^2)\}^{1/2} \tag{31}$$

and

$$\gamma = \Gamma T.$$

$Q(s)$ has simple poles at

$$s_k = -\left(\frac{\gamma^2+y_k^2}{2\gamma\langle E\rangle}\right) \tag{33}$$

y_k being the positive roots of the transcendental equation

$$\tan y_k = \frac{2\gamma y_k}{y_k^2-\gamma^2}. \tag{34}$$

3. The probability distribution

The probability distribution of E is the inverse Laplace transform of the moment generating function

$$P(E) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} Q(s)\,\mathrm{e}^{Es}\,ds. \tag{35}$$

We evaluate this integral by making use of the fact that the poles of $Q(s)$ are simple and hence the residues $R(s_k)$ of the integrand are

$$R(s_k) = \mathrm{e}^{Es_k}\left(\frac{dQ^{-1}}{ds}\right)^{-1}_{s=s_k}. \tag{36}$$

The poles all occur on the negative real axis of s. Thus

$$P(E) = \sum_k R(s_k) = \sum_k \mathrm{e}^{Es_k}\left(\frac{dQ^{-1}}{ds}\right)^{-1}_{s=s_k} \tag{37}$$

the sum being taken over all positive roots of equation (31). At each pole of $Q(s)$ we have, of course,

$$Q^{-1}(s_k) = \left\{\cos y_k + \left(\frac{\gamma}{2y_k}-\frac{y_k}{2\gamma}\right)\sin y_k\right\}\mathrm{e}^{-\gamma} = 0. \tag{38}$$

From equations (30) and (33),

$$\frac{dQ^{-1}(s)}{ds} = -\mathrm{e}^{-\gamma}\left\{\left(\frac{\gamma}{2y}-\frac{y}{2\gamma}\right)\cos y - \left(1+\frac{1}{2\gamma}+\frac{\gamma}{2y^2}\right)\sin y\right\}\frac{\langle E\rangle\gamma}{y}. \tag{39}$$

Using the factored form of equation (31)

$$y_k \tan \tfrac{1}{2}y_k = \gamma \tag{40a}$$

$$y_k \cot \tfrac{1}{2}y_k = -\gamma \tag{40b}$$

together with equations (38) and (39), we find that

$$\left\{\frac{dQ^{-1}(s)}{ds}\right\}^{-1}_{s=s_k} = \pm\frac{\mathrm{e}^{\gamma}}{\langle E\rangle}\frac{2y_k^2}{y_k^2+2\gamma+\gamma^2} = \pm\frac{\mathrm{e}^{\gamma}}{2\langle E\rangle}\frac{d(y_k^2)}{d\gamma} \tag{41}$$

where the positive or negative sign is chosen according as y_k is a solution of (40*a*) or (40*b*) respectively. Substituting equations (41) into equation (37) and ordering the terms such that $y_k < y_{k+1}$ and y_0 is the first positive root

$$P(E) = \frac{\mathrm{e}^{\gamma}}{\langle E\rangle}\sum_{k=0}^{\infty}(-1)^k\frac{2y_k^2}{y_k^2+2\gamma+\gamma^2}\exp\left\{-\left(\frac{\gamma^2+y_k^2}{2\langle E\rangle\gamma}\right)E\right\}. \tag{42}$$

For reasonable values of the parameters the series (42) is rapidly covergent. Indeed, in most cases the first two terms give a very accurate result.

4. Asymptotic forms

4.1. $\gamma \ll 1$

In this limit the series (42) for $P(E)$ is dominated by its first term, corresponding to the smallest root of equation (40*a*):

$$y^2 \sim 2\gamma. \tag{43}$$

Since all other solutions of (40*a*) and (40*b*) remain non-zero as $\gamma \to 0$, succeeding terms vanish as $\exp(-y_n{}^2/\gamma)$. Substituting (43) into the first term of (42) leads to

$$P(E) = \frac{1}{\langle E\rangle}\frac{4}{4+\gamma}\exp\left\{\gamma - \frac{E(\gamma+2)}{2\langle E\rangle}\right\} \tag{44a}$$

and expanding this expression to first order in γ we obtain

$$P(E) \simeq \frac{1}{\langle E\rangle}\left\{1+\left(3-\frac{2E}{\langle E\rangle}\right)\frac{\gamma}{4}\right\}\exp\left(-\frac{E}{\langle E\rangle}\right). \tag{44b}$$

As $\gamma \to 0$ (44*b*) reduces to the asymptotic limit (Mandel and Wolf 1965, equation 4.39)

$$P(E) = \frac{1}{\langle E\rangle}\exp\left(-\frac{E}{\langle E\rangle}\right).$$

4.2. $\gamma \gg 1$

For large but finite values of γ the roots of (40*a*) and (40*b*) are multiples of π except when $y_k \sim \gamma$. In this case equation (42) may be written approximately as

$$P(E) \simeq \frac{1}{\langle E\rangle}\exp\left\{\gamma\left(1-\frac{E}{2\langle E\rangle}\right)\right\}\sum_n (-1)^n \exp\left(-\frac{n^2\pi^2 E}{2\langle E\rangle\gamma}\right). \tag{45a}$$

We have averaged the pre-factor multiplying the exponential terms in (42) and neglected the contribution of the region where $y_n \ll \gamma$. The sum appearing in (45) is closely related to the theta function $\theta_3(\pi/2 : \pi E/2\langle E\rangle\gamma)$ and for large γ is well approximated by $(\langle E\rangle\gamma/2\pi E)^{1/2}\exp(-\langle E\rangle\gamma/2E)$ (Jeffreys and Jeffreys 1946, p. 48) so that

$$P(E) \simeq \left(\frac{\eta}{\pi}\right)^{1/2}\exp\{-\eta(E-\langle E\rangle)^2\} \tag{45b}$$

where $\eta = \gamma/2E\langle E\rangle$. In the limit $\eta \to \infty$ this becomes the Dirac delta function $\delta(E-\langle E\rangle)$. The generating function corresponding to the distribution (45*b*) is of the form

$$G(s) = \frac{\gamma}{z}\exp(\gamma - z) \tag{46}$$

where

$$z = (2\langle E\rangle\gamma s + \gamma^2)^{1/2}.$$

This differs from Glauber's result for large γ (Glauber 1965, equation 17.56) by the presence of the factor γ/z.

4.3. $\gamma \sim 1$

When γ is of the order of unity our calculations have shown that $P(E)$ is well approximated, particularly over the region where it is decreasing ($E/\langle E\rangle > 0{\cdot}4$), by the first two terms of the series (42), corresponding to the first roots of (40*a*) and (40*b*). We thus have the approximate formula

$$P(E) = \frac{1}{\langle E\rangle}\exp\left\{\gamma\left(1-\frac{E}{2\langle E\rangle}\right)\right\}\sum_{n=1,2}(-1)^{n-1}\frac{2y_n{}^2}{y_n{}^2+\gamma^2+2\gamma}\exp\left(-\frac{y_n{}^2 E}{2\langle E\rangle\gamma}\right) \tag{47}$$

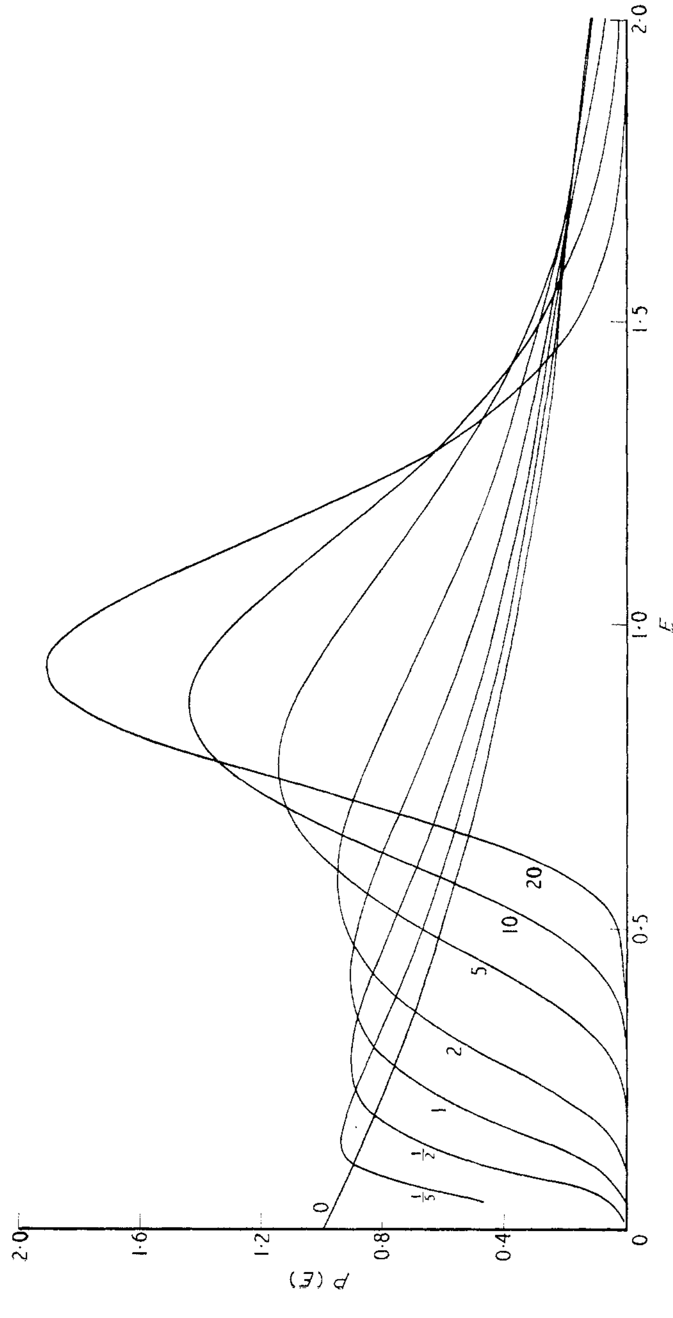


Figure 1. The probability distribution of $E(T)$ for the values of ΓT shown.

where

$$y_1 \tan \tfrac{1}{2} y_1 = \gamma \qquad 0 < y_1 < \pi \tag{48a}$$

$$y_2 \cot \tfrac{1}{2} y_2 = -\gamma \qquad \pi < y_2 < 2\pi. \tag{48b}$$

For small γ equation (47) is more complicated than, but rather better than, equation (44*b*). For this case the roots of equations (48*a*) and (48*b*) can be taken as $y_1 = (2\gamma)^{1/2}$ and $y_2 = \pi$.

5. The distribution for arbitrary linewidth

The series for $P(E)$ of equation (42) was computed for a set of values of γ ranging from 0·2 through 1·0 up to 20. The first 25 pairs of roots of equation (31) were computed and stored and the summation was carried out by adding pairs of terms at a time until a specified small incremental value was reached; the summation was then stopped and the number of terms required together with the value of $P(E)$ printed out. The mean $\langle E \rangle$ was put equal to unity, for general values of $\langle E \rangle$ the ordinates are $\langle E \rangle P(E\langle E \rangle)$.

The probability distributions are plotted in figure 1. Their general shape may be compared with the results of Slepian (1958, figure 1) for the scalar problem, which, as we pointed out above, would correspond to a detector which responded to the classical field. Both sets of curves tend to exponentials at small γ and to a δ function on unity at large γ but the detailed behaviour is quite different. In the limit as γ tends to zero the quantum noise has a finite probability at the origin while the classical probability curve goes to infinity there. At the other limit of large γ, corresponding to the coherence time of the source much less than the integration time, the quantum noise curves peak up rather more rapidly than the classical ones.

The accuracy asked for in the results shown was for $P(E)$ to differ by less than 0·001 from its previous value upon the addition of the next pair of terms. For this accuracy at $\gamma = 0{\cdot}2$ a single pair of terms was sufficient for all values of E down to 0·05. For $\gamma = 1{\cdot}0$ two pairs of terms were needed below 0·35 and as γ was increased to 20, more and more terms were needed for low values of E. For $\gamma = 20$, seven pairs of terms were required at 0·55, and for E as low as 0·05, at this same value of γ, twenty-four pairs of terms were used.

The experimental verification of these distributions would require the inversion of the corresponding photon-counting distributions $p(n, T)$, given by (Mandel 1959)

$$p(n, T) = \int_0^\infty \frac{(\alpha E)^n}{n!} \exp(\alpha E) P(E)\, dE \tag{49}$$

where α is the quantum efficiency of the detector, since $P(E)$ is not directly available experimentally. This inversion has been discussed by Wolf and Mehta (1964) but a practical procedure is to compare moments as discussed below.

The photon-counting distribution for the problem we have considered is a function of $\alpha\langle E \rangle$ and γ only, as can be seen from equations (47) and (49). The former quantity is the measured mean number of counts per sample time, say $\bar{n}$. In figures 2(*a*), 2(*b*), 2(*c*) and 2(*d*) we give computed examples of photon-counting distributions for various values of γ and $\bar{n}$.

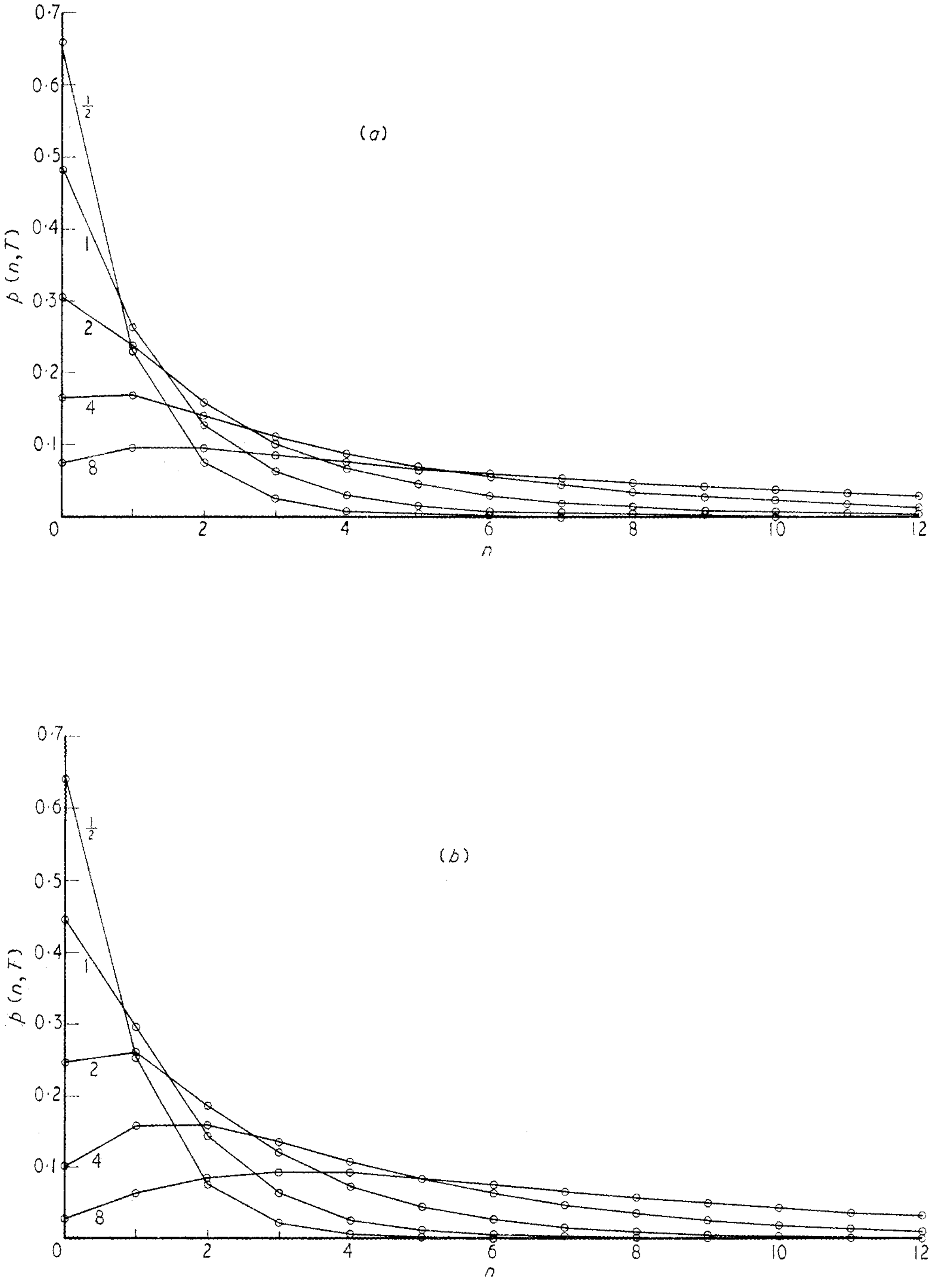
The single feature of each distribution $P(E)$ of most importance, if one can be sure that the light is Gaussian and the spectrum Lorentzian, is its variance or second central moment which can be shown directly by the method of Rice (1948, p. 88) to be given by

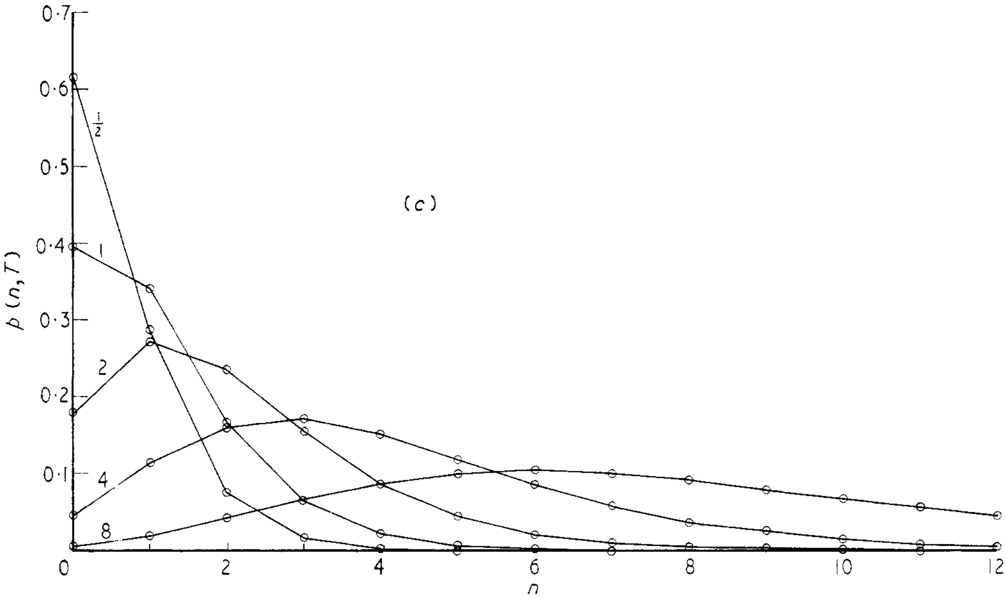
$$\sigma_T{}^2 = \frac{\langle E \rangle^2}{2\gamma^2} (\mathrm{e}^{-2\gamma} + 2\gamma - 1). \tag{50}$$

The moments of $P(E)$ are proportional to the factorial moments of $p(n)$; these are given by the formula (Glauber 1965)

$$\sum_n n(n-1) \ldots (n-k+1) p(n) = \left. \frac{d^k G(s)}{ds^k} \right|_{s=0}$$

and have been evaluated by Bédard (1966). Our expression (50) for the mean-square noise is a simple analytic form which can also be found from his recurrence relations.


(a)
p (n, T)
0·7
0·6
0·5
0·4
0·3
0·2
0·1
1/2
1
2
4
8
0
2
4
6
8
10
12
n
(b)
p (n, T)
0·7
0·6
0·5
0·4
0·3
0·2
0·1
1/2
1
2
4
8
0
2
4
6
8
10
12
n



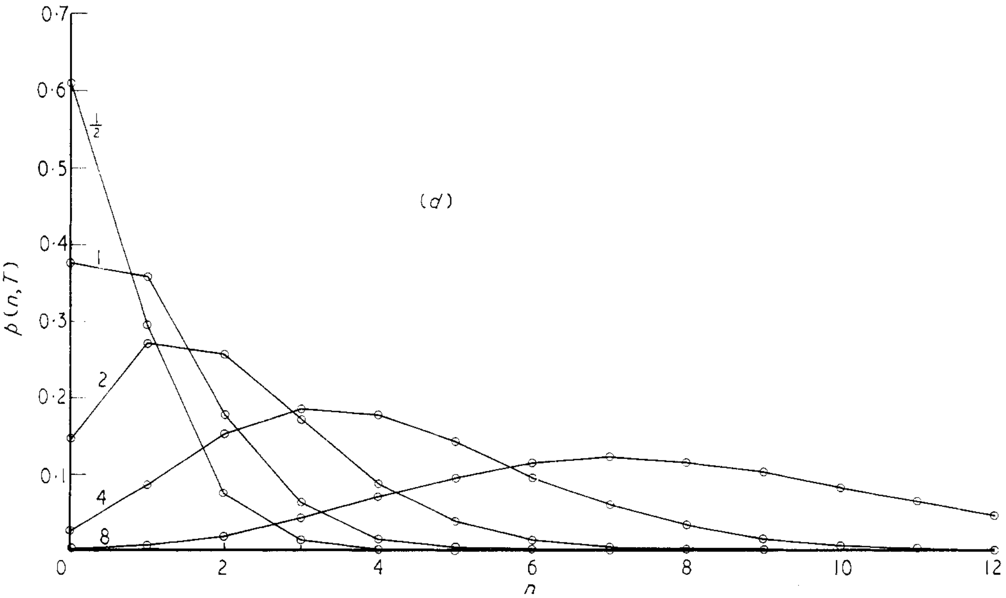


Figure 2. The photon-counting distributions for the values of $\bar{n}$ shown
(*a*) $\Gamma T = 0{\cdot}2$
(*b*) $\Gamma T = 1{\cdot}0$
(*c*) $\Gamma T = 5{\cdot}0$
(*d*) $\Gamma T = 20{\cdot}0$
These histograms are shown as linked curves for clarity.

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